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Mathematical Modeling for Flood Mitigation: Effect of Bifurcation Angles in River Flowrates

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Abstract This paper investigates the river flowrate at two branches of bifurcated river. The mathematical model from the literature is formulated based on momentum principle and mass continuity to cope with river flowrate at different bifurcation angles. The hydraulic variables, geometric properties of trapezoidal cross-sectional river and other physical characteristics of bifurcated river are provided, which may be assumed to be given beforehand for practical applications. An example of river bifurcation problem is given by UTM Centre for Industrial and Applied Mathematics (UTM-CIAM), Universiti Teknologi Malaysia. Maple software is used to implement the proposed model equation and generate the results. The amount of bifurcated river flowrate with different bifurcation angles is determined, resulting in a reasonable discussion. It is shown that for specific bifurcation angles, the river flowrates after the bifurcated junction are less than the critical flowrate. Finally, the results of applied problem indicate that the right-angled river bifurcation would be preferable to mitigate flood.

Keywords Bifurcation, Flowrate, Momentum Principle

1. Introduction

River bifurcation is the process that determines the distribution of flow, sediments and contaminants along the downstream river branches. This process is important in order to mitigate flood due to climate change. There have been several approaches in investigating the river bifurcation or bifurcated open-channel flow. For instance, [1] used both analytical and experimental ways to study the bifurcated open-channel flow. The channels used are of rectangular cross-sectional and the branch channel being set at a right-angled midway along the straight main channel. The estimation of the flowrate ratio in terms of the Froude number and the depth ratio had been obtained using theoretical model in [2]. The authors provided the experimental data for the validity of their proposed model.

Based on experimental observations, the work of [3] carried out a study on depth discharge relationship and energy-loss coefficient for a subcritical, equal-width, right-angled dividing subcritical flow over a horizontal bed in a narrow aspect ratio channel. The theoretical model for subcritical flows in dividing open channel junction is derived in [4] with the aid of the overall mass conservation together with the momentum principle in the streamwise direction to two control volumes through the junction. Further, a physical model with meandering features is constructed in [5] to investigate the effect of off-take angles on the flow distribution at a concave channel bifurcation.

A theoretical model for predicting depth of water with certain dividing angles has been proposed by [6]. The authors developed the model equations for both combining and dividing types of subcritical flows at channel junctions using the principal of momentum balance. The width of all the channels both in case of combining and dividing has been kept differently.

An unsteady mathematical model for predicting flow divisions at a right-angled open-channel junction [7] and hydrodynamic model [8, 9] for bifurcating stream was also done. More recently, the findings of nearly 10 years of researches into modeling bifurcation system with numerous simulation techniques have been reviewed [10].

However, none of the above work analyzes the effect of different bifurcation angles in river flowrate. In fact, the majority of the existing models are designed for a right-angled junction. Therefore, the aim of this paper is to investigate the behavior of river flowrates influenced by different bifurcation angles using mathematical model approach. The following section will describe the characteristics of the bifurcated channel and its geometric properties. Section 3 deals with the formulation of the mathematical model. An example of river bifurcation problem is given in Section 4. In section 5, the results are analyzed and discussed. Finally, some conclusions are made as well as the recommendation for future study.

2. Methodology

This section provides the detailed description of the channel and formulation of the model for the present study.

2.1. Description of the Channel

The characteristics of the bifurcated open-channel and its cross-sectional properties have to be considered for the equation of mathematical model. The schematic layout of the bifurcated channel is illustrated in Figure 1. A main channel is connected with two branch channels. The angles, θ_1 and θ_2 at the bifurcated junction are called bifurcation angles. For the application of momentum conservation law, we consider the boundaries of control volume as shown by the dotted line. The section has been positioned at the distance of two times the width of the channel at upstream and three times the width of the channel at downstream of the bifurcation.

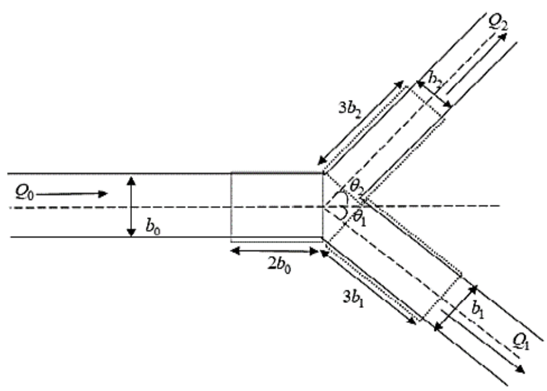

Figure 1. Schematic layout of the bifurcated channel: Q = flowrate, b = bottom width of channel, θ = bifurcation angles, 0 = main channel, 1 = channel 1, 2 = channel 2

The channels are assumed to be uniform cross section. Channel cross sections can be considered to be either regular or irregular. A regular section is one whose shape does not vary along the length of the channel, whereas an irregular section will have changes in its geometry. The most common irregular section of open channel is a trapezoidal shape. The typical trapezoidal cross-sectional is shown in Figure 2.

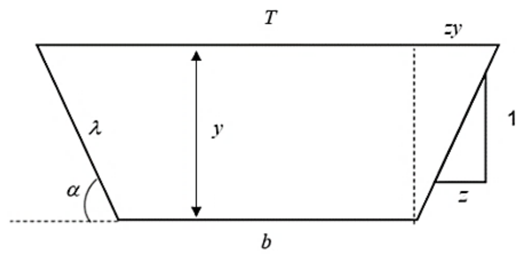

Figure 2. Geometric details of the typical trapezoidal cross-sectional: α = angle of the slope side, b = bottom width, y = depth of flow, λ = wetted length measured along the slope side, T = top width, z = channel side slope

According to [11], flow hydraulics and momentum exchange in straight channels are significantly influenced by geometric and hydraulic variables. The cross-sectional area, A is given by $A = by + zy^2$, in which b is the width of the channel bottom and y is the depth of flow. The side slope is usually specified as horizontal : vertical, $z:1$. Additional parameters for open channel flow are the wetted perimeter, P_W, the hydraulic radius, R_H and the hydraulic depth, D. The wetted perimeter, $P_W = b + 2\lambda$ is the length of the line of contact between the water and the channel where the wetted length measured along the slope side is given by $\lambda = \sqrt{y^2 + (yz)^2}$.

The hydraulic radius, R_H is the area divided by the wetted perimeter, that is, $R_H = \frac{A}{P_W}$. The hydraulic depth is the area divided by the top width, $D = \frac{A}{T}$ where $T = b + 2zy$. Flow area is the cross-sectional area of the flow taken perpendicular to the flow direction.

Even though the river cross-sectional areas of the main channel and channel 1 are assumed to be similar, the capability of the channel to convey water can vary due to bifurcation angle. If the flowrate is unknown, a uniform velocity, V that applies to an entire cross-sectional can be determined using Manning's equation [12, 13] as shown below:

$$V = \frac{1}{n} R_H{}^{0.66} S^{0.5},$$

where n is the roughness coefficient and S is the average slope of channel. The dimensionless ratio of the inertial forces to gravitational forces acting on the flow is represented by Froude number, F is defined as

$$F = \frac{V}{\sqrt{gD}} = \frac{Q}{A\sqrt{g\left(\frac{A}{T}\right)}},$$

where V is the velocity, D is the hydraulic depth and the gravitational acceleration, g is 9.80665m/s^2. The Froude number plays a significant role in open channel flow analysis. The hydraulic behavior of channel flow varies significantly depending on whether the flow is critical ($F=1$), subcritical ($F<1$) or supercritical ($F>1$).

The division of flowrate at bifurcated channel can be determined using the aid of momentum principle and mass continuity with the following assumptions:

(a) Main channel is straight prismatic channel, to which two branches of bifurcated junction are connected. All channels are trapezoidal cross-sectional.
(b) The flow is from main channel into channels 1 and 2.
(c) The velocities and water surface elevations are constant across the channels at the inflow and outflow sections of the control volumes.
(d) The pressure distribution is hydrostatic at all sections of control volume.
(e) The geometrical properties such as channels width, channels depth, control volume lengths and slope of channel are known.
(f) The depth of flow in the main channel, channels 1 and 2 are equal.
(g) The shear stresses on the flow surface due to wind, the effects of vertical acceleration and the wall friction force as compared to other forces are neglected.

2.2. Formulation of the Model

This section describes the detail formulation of mathematical model based on momentum principle and mass continuity [7]. The basic continuity equation is taken as starting point for the formulation,

$$Q_0 = Q_1 + Q_2, \tag{1}$$

where $Q_0 = A_0V_0$, $Q_1 = A_1V_1$ and $Q_2 = A_2V_2$. The terms Q_0, Q_1 and Q_2 are flowrates, A_0, A_1 and A_2 are trapezoidal cross-sectional areas while V_0, V_1 and V_2 are velocities in main channel, channel 1 and channel 2 respectively. The hydrostatic force on the horizontal strip of A will be $P=\gamma A$ where $A=by+zy^2$ and γ are the specific weight of water. Therefore, the total horizontal force can be determined as follows:

$$P=\gamma\int_0^y by+zy^2\, dA=\left[\frac{by^2}{2}+\frac{zy^3}{3}\right]_0^y=\gamma\left(\frac{by^2}{2}+\frac{zy^3}{3}\right). \tag{2}$$

By applying the continuity equation (1) and momentum principle in the flow direction of the main channel, we obtain

$$\begin{aligned} &P_0-P_2\cos\theta_2-P_1\cos\theta_1-U_2-U_1-\Delta P= \\ &\frac{\gamma}{g}\left(Q_2V_2\cos\theta_2+Q_1V_1\cos\theta_1-Q_0V_0\right). \end{aligned} \tag{3}$$

The terms of momentumtransfering from the main channel to the branch channels are given in the following forms [7]:

$$U_1=\rho Q_1V_0C\sin\theta_1,\ U_2=\rho Q_2V_0C\sin\theta_2,$$

where

$$C=\frac{5}{6}-\frac{F_0{}^2}{40}-\frac{k_0}{12q_r}\left(\frac{1+2k_0}{(1+k_0)^2}\right).$$

Noting that $\Delta P=\gamma\left(\frac{b_0y_2{}^2}{2}+\frac{zy_2{}^3}{3}\right)-P_2\cos\theta_2$ while the density of water, ρ is related to γ and g which can be determined as $\rho=\frac{\gamma}{g}$. By moving the terms, U_1 and U_2 to the right hand side (RHS), (3) can be written as follows:

$$\begin{aligned} &P_0-P_2\cos\theta_2-P_1\cos\theta_1-\Delta P=\frac{\gamma}{g}\left[Q_2V_2\cos\theta_2\right. \\ &\left.+Q_1V_1\cos\theta_1-Q_0V_0+\rho Q_2V_0C\sin\theta_2+\rho Q_1V_0C\sin\theta_1\right]. \end{aligned} \tag{4}$$

By taking the left hand side (LHS) of (4), we have the following equation:

$$P_0-P_2\cos\theta_2-P_1\cos\theta_1-\Delta P\,. \tag{5}$$

Based on (2), we simplify (5) as follows:

$$\begin{aligned} &\gamma\left[\frac{1}{2}\left(b_0y_0{}^2-b_0y_2{}^2-b_1y_1{}^2\cos\theta_1\right)\right. \\ &\left.+\frac{z}{3}\left(y_0{}^3-y_2{}^3-y_1{}^3\cos\theta_1\right)\right]. \end{aligned} \tag{6}$$

Now taking the RHS of (4),

$$\begin{aligned} &\frac{\gamma}{g}\left[Q_2V_2\cos\theta_2+Q_1V_1\cos\theta_1-Q_0V_0\right. \\ &\left.+\rho Q_2V_0C\sin\theta_2+\rho Q_1V_0C\sin\theta_1\right]. \end{aligned} \tag{7}$$

Based on (1), we produce

$$\begin{aligned} &\frac{\gamma}{g}\left[\frac{Q_2Q_2}{A_2}\cos\theta_2+\frac{Q_1Q_1}{A_1}\cos\theta_1-\frac{Q_0Q_0}{A_0}\right. \\ &\left.+\frac{Q_2Q_0}{A_0}C\sin\theta_2+\frac{Q_1Q_0}{A_0}C\sin\theta_1\right]. \end{aligned} \tag{8}$$

By using algebraic manipulation in (8), yields

$$=\frac{\gamma Q_0^2}{gA_0}\left[\frac{1}{Q_0^2}\frac{Q_2^2}{A_2/A_0}\cos\theta_2+\frac{1}{Q_0^2}\frac{Q_1^2}{A_1/A_0}\cos\theta_1 -1+C\left(\frac{Q_2}{Q_0}\sin\theta_2+\frac{Q_1}{Q_0}\sin\theta_1\right)\right]. \quad (9)$$

Let the flowrate ratio, $q_r=\frac{Q_1}{Q_0}$. Based on (1) and $Q_1=Q_0q_r$, we get $Q_0=Q_2+Q_0q_r$ and subsequently produce $\frac{Q_2}{Q_0}=1-q_r$. Therefore, the following equation is obtained:

$$\frac{\gamma Q_0^2}{gA_0}\left[\frac{(1-q_r)^2}{A_2/A_0}\cos\theta_2+\frac{q_r^2}{A_1/A_0}\cos\theta_1 -1+C\left((1-q_r)\sin\theta_2+q_r\sin\theta_1\right)\right]. \quad (10)$$

Knowing that

$$\frac{Q_0^2}{gA_0}=\frac{Q_0^2}{gA_0}\times\frac{T_0}{T_0}\times\frac{A_0^2}{A_0^2}=\frac{Q_0^2T_0}{gA_0^3}\frac{A_0^2}{T_0}=F_0^2\frac{A_0^2}{T_0}.$$

Subsequently, yielding

$$F_0^2\frac{b_0^2y_0^2+2b_0zy_0^3+z^2y_0^4}{b_0+2zy_0}. \quad (11)$$

Let $z=\frac{k_0b_0}{y_0}$ and factorize, we have

$$F_0^2\frac{b_0^2y_0^2\left(1+2k_0+k_0^2\right)}{b_0\left(1+2k_0\right)}. \quad (12)$$

Hence, $\frac{Q_0^2}{gA_0}=F_0^2b_0y_0^2\frac{(1+k_0)^2}{(1+2k_0)}$. The term $\frac{A_2}{A_0}$ can be written as follows:

$$\frac{A_2}{A_0}=\frac{b_2y_2+zy_2^2}{b_0y_0+zy_0^2}. \quad (13)$$

Let $\frac{y_2}{y_0}=y_r$, $\frac{b_2}{b_0}=Br_2$ and $\frac{b_1}{b_0}=Br_1$, we get

$$\frac{A_2}{A_0}=\frac{b_2y_2+k_0b_0y_2y_r}{b_0y_0+k_0b_0y_0}. \quad (14)$$

Similarly, the term $\frac{A_1}{A_0}$ is given by

$$\frac{A_1}{A_0}=\frac{b_1y_1+\frac{k_0b_0y_1^2}{y_0}}{b_0y_0+\frac{k_0b_0y_0^2}{y_0}}. \quad (15)$$

In our case, it has to be noted that the depths of all channels are equal, $y_0=y_1=y_2$. Thus, the depth ration is given by $y_r=\frac{y_1}{y_0}=\frac{y_2}{y_0}$. Equation (15) is written as follows:

$$\frac{A_1}{A_0}=\frac{b_1y_1+k_0b_0y_1y_r}{b_0y_0+k_0b_0y_0}. \quad (16)$$

By multiplying the numerator and denominator of (14) and (16) with $\frac{1}{b_0y_0}$, we obtain

$$\frac{A_1}{A_0}=\frac{\left(Br_1+k_0y_r\right)y_r}{1+k_0}, \quad (17)$$

$$\frac{A_2}{A_0}=\frac{\left(Br_2+k_0y_r\right)y_r}{1+k_0}. \quad (18)$$

We substitute (17) and (18) into (10), yielding

$$\gamma F_0^2b_0y_0^2\frac{(1+k_0)^2}{(1+2k_0)}\left[\frac{(1-q_r)^2}{\left(Br_2+k_0y_r\right)y_r/(1+k_0)}\cos\theta_2 +\frac{q_r^2}{\left(Br_1+k_0y_r\right)y_r/(1+k_0)}\cos\theta_1-1 +C\left((1-q_r)\sin\theta_2+q_r\sin\theta_1\right)\right]. \quad (19)$$

Simplifying (19), we get

$$\gamma F_0^2b_0y_0^2\frac{(1+k_0)^2}{(1+2k_0)}\left[\frac{1+k_0}{y_r}\left(\frac{(1-q_r)^2}{\left(Br_2+k_0y_r\right)}\cos\theta_2 +\frac{q_r^2}{\left(Br_1+k_0y_r\right)}\cos\theta_1\right)-1+C\left((1-q_r)\sin\theta_2+q_r\sin\theta_1\right)\right]. \quad (20)$$

Finally, LHS of (6) is equal to RHS of (20) that becomes

$$\frac{(1+2k_0)}{b_0y_0^2}\left[\frac{1}{2}\left(b_0y_0^2-b_0y_2^2-b_1y_1^2\cos\theta_1\right) +\frac{z}{3}\left(y_0^3-y_2^3-y_1^3\cos\theta_1\right)\right]=F_0^2\left(1+k_0\right)^2 \left[\frac{1+k_0}{y_r}\left(\frac{(1-q_r)^2}{\left(Br_2+k_0y_r\right)}\cos\theta_2+\frac{q_r^2}{\left(Br_1+k_0y_r\right)}\cos\theta_1\right) -1+C\left((1-q_r)\sin\theta_2+q_r\sin\theta_1\right)\right]. \quad (21)$$

After simplification of (21), the general equation of bifurcated flow is obtained in the following form:

$$\begin{aligned}&(1+2k_0)\left[\frac{1}{2}\left(1-y_r{}^2-Br_1y_r{}^2\cos\theta_1\right)\right.\\&\left.+\frac{k_0}{3}\left(1-y_r{}^3-y_r{}^3\cos\theta_1\right)\right]=F_0{}^2(1+k_0)^2\left[\frac{1+k_0}{y_r}\right.\\&\left(\frac{(1-q_r)^2}{(Br_2+k_0y_r)}\cos\theta_2+\frac{q_r{}^2}{(Br_1+k_0y_r)}\cos\theta_1\right)\\&\left.-1+C\left((1-q_r)\sin\theta_2+q_r\sin\theta_1\right)\right].\end{aligned} \tag{22}$$

3. River Bifurcation Problem

In this section, we give special attention to river bifurcation problem in Sungai Nenggiri, Gua Musang, Kelantan. Sungai Nenggiri that is geographically located in the north eastern part of Peninsular Malaysia within latitude 4.97024° to 4.96951° North and 101.77144° to 101.77207° East. This river is considered in this study due to serious floods' occurrence during Monsoon season in the past few years. Extensive flooding throughout the catchment occurs during heavy and prolonged rainfall resulting in high river flow. The river flow will overspill the banks of Sungai Nenggiri, disrupting road network and human life.

The main mitigation action that can be taken is by diverting some of Sungai Nenggiri's flow during peak flow to a new river, namely Sungai Anak Nenggiri. The amounts of river flow from Sungai Nenggiri (main channel) going through Sungai Nenggiri after the bifurcation junction (channel 1) and Sungai Anak Nenggiri (channel 2) are depending on the bifurcation angles, θ_1 and θ_2. The flowrate in the main channel is assumed to be $Q_0 = 1000\text{m}^3/\text{s}$ while Q_1 and Q_2 are the flowrates in channel 1 and channel 2 respectively. The critical flowrate in channel 1 is expected to be $Q_1 = 800\text{m}^3/\text{s}$. We theorize that if the flowrate exceeds this value, flood will occur in channel 1. Therefore, (22) can be applied to determine the amount of flowrates in channel 1 and channel 2 with different bifurcation angles.

This general problem is given by UTM Centre for Industrial and Applied Mathematics (UTM-CIAM), Universiti Teknologi Malaysia. However, the real experimental data of Sungai Nenggiri is unavailable at this time for error analysis. For application purpose, we assume that the channels are normal, clean, straight, full stage, with no rifts or deep pools. Thus, $n = 0.03$ is selected as manning's coefficient while the slope of the main channel is $S = 0.0001814260235\text{m}$. Since the Froude number for the main channel is $F_0 = 0.174902437$, it can be said that the flow is subcritical. The geometric and hydraulic properties of bifurcated channel used in the proposed model are presented in Table 1.

Table 1. Geometric and hydraulic properties (GHP) of bifurcated channel

GHP	Main channel	Channel 1	Channel 2
α	60^0	60^0	60^0
y	3.5m	3.5m	3.5m
z	4.081632657m	4.081632657m	0.859291084m
T	300m	300m	60.15037594m
b	271.285714m	271.285714m	54.13533835m
A	1000m^2	1000m^2	200m^2
λ	14.70821651m	14.70821651m	4.614668923m
P_w	300.8450044m	300.8450044m	63.3646762m
R_H	3.323970767m	3.323970767m	3.156332708m
D	3.33m	3.33m	3.325m

4. Results

To analyze the results, the model (22) is performed using Maple software. The bifurcation angles, θ_1 and θ_2 considered in this study are 0^0, 15^0, 30^0, 45^0, 60^0, 75^0 and 90^0. The values of flowrate ratios, q_r, flowrates in channel 1, Q_1 and flowrates in channel 2, Q_2 are tabulated in Tables 2-8. It has to be mentioned that q_r is the ratio of flowrate in channel 1 to the flowrate in main channel. For simplicity, the graphical representations of flowrate ratios and bifurcation angles are shown in Figure 3.

Table 2. Flowrates in channels 1 and 2 when $\theta_1 = 0^0$

θ_1	θ_2	q_r	Q_1, m^3/s	Q_2, m^3/s
0	0	0.8067940552	806.7940552	193.2059448
0	15	0.8227092780	822.7092780	177.2907220
0	30	0.8284124759	828.4124759	171.5875241
0	45	0.8214011067	821.4011067	178.5988933
0	60	0.7928048911	792.8048911	207.1951089
0	75	0.7125371460	712.5371460	287.4628540
0	90	0.4161533812	416.1533812	583.8466188

Table 3. Flowrates in channels 1 and 2 when $\theta_1 = 15^0$

θ_1	θ_2	q_r	Q_1, m^3/s	Q_2, m^3/s
15	0	0.7911862451	791.1862451	208.8137549
15	15	0.8067650940	806.7650940	193.2349060
15	30	0.8110586289	811.0586289	188.9413711
15	45	0.8010471671	801.0471671	198.9528329
15	60	0.7663678231	766.3678231	233.6321769
15	75	0.6717559187	671.7559187	328.2440813
15	90	0.3192905923	319.2905923	680.7094077

Table 4. Flowrates in channels 1 and 2 when $\theta_1 = 30^0$

θ_1	θ_2	q_r	Q_1, m^3/s	Q_2, m^3/s
30	0	0.7869497049	786.9497049	213.0502951
30	15	0.8027210236	802.7210236	197.2789764
30	30	0.8067320739	806.7320739	193.2679261
30	45	0.7957057968	795.7057968	204.2942032
30	60	0.7582902466	758.2902466	241.7097534
30	75	0.6546484797	654.6484797	345.3515203
30	90	0.2401904453	240.1904453	759.8095547

Table 5. Flowrates in channels 1 and 2 when $\theta_1 = 45^0$

θ_1	θ_2	q_r	Q_1, m^3/s	Q_2, m^3/s
45	0	0.7949051046	794.9051046	205.0948954
45	15	0.8114393561	811.4393561	188.5606439
45	30	0.8164352045	816.4352045	183.5647955
45	45	0.8066871811	806.6871811	193.3128189
45	60	0.7705429480	770.5429480	229.4570520
45	75	0.6645994258	664.5994258	335.4005742
45	90	0.1722446834	172.2446834	827.7553166

Table 6. Flowrates in channels 1 and 2 when $\theta_1 = 60^0$

θ_1	θ_2	q_r	Q_1, m^3/s	Q_2, m^3/s
60	0	0.8159655445	815.9655445	184.0344555
60	15	0.8339066727	833.9066727	166.0933273
60	30	0.8414225983	841.4225983	158.5774017
60	45	0.8358869677	835.8869677	164.1130323
60	60	0.8066091461	806.6091461	193.3908539
60	75	0.7097821934	709.7821934	290.2178066
60	90	0.1112785518	111.2785518	888.7214482

Table 7. Flowrates in channels 1 and 2 when $\theta_1 = 75^0$

θ_1	θ_2	q_r	Q_1, m^3/s	Q_2, m^3/s
75	0	0.8509646114	850.9646114	149.0353886
75	15	0.8710508212	871.0508212	128.9491788
75	30	0.8830122328	883.0122328	116.9877672
75	45	0.8856383590	885.6383590	114.3616410
75	60	0.8716827337	871.6827337	128.3172663
75	75	0.8063916606	806.3916606	193.6083394
75	90	0.0542840796	54.28407957	945.7159204

Table 8. Flowrates in channels 1 and 2 when $\theta_1 = 90^0$

θ_1	θ_2	q_r	Q_1, m^3/s	Q_2, m^3/s
90	0	0.6742997042	674.2997042	325.7002958
90	15	0.7031325671	703.1325671	296.8674329
90	30	0.7164798741	716.4798741	283.5201259
90	45	0.7131229768	713.1229768	286.8770232
90	60	0.6837385036	683.7385036	316.2614964
90	75	0.5844868749	584.4868749	415.5131251
90	90	0.8198591028	819.8591028	180.1408972

The amount of Q_1 is less than $800m^3/s$ when $\theta_1 = 0^0$ and $\theta_2 = 60^0$, 75^0 or 90^0. If $\theta_1 = 30^0$, Q_1 is less than $800m^3/s$ except when $\theta_2 = 15^0$ or 30^0. For $\theta_1 = 15^0$ or 45^0, Q_1 is less than $800m^3/s$ when $\theta_2 = 0^0$, 60^0, 75^0 or 90^0. It also can be observed that Q_1 is less than $800m^3/s$ when $\theta_1 = 60^0$ and $\theta_2 = 75^0$ or 90^0. From Tables 2-7, it is observed that Q_1 decreases significantly when $\theta_2 = 90^0$. The lowest value of Q_1 is when $\theta_1 = 75^0$ and $\theta_2 = 90^0$ as presented in Table 7. However, Q_1 becomes greater than the critical flowrate when $\theta_1 = \theta_2 = 90^0$. The values of bifurcation angle when Q_1 is less than the critical flowrate that is summarized in Table 9.

Table 9. Bifurcation angles when $q_r < 0.8$

θ_1	θ_2
0^0	60^0, 75^0, 90^0
15^0	0^0, 60^0, 75^0, 90^0
30^0	0^0, 45^0, 60^0, 75^0, 90^0
45^0	0^0, 60^0, 75^0, 90^0
60^0	75^0, 90^0
75^0	90^0
90^0	0^0, 15^0, 30^0, 45^0, 60^0, 75^0

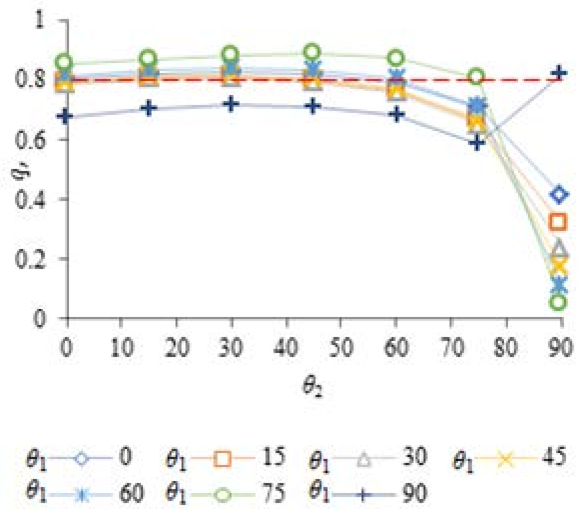

Figure 3. Graph of flowrate ratios, q_r with respect to θ_2 when $\theta_1 = 0^0, 15^0, 30^0, 45^0, 60^0, 75^0, 90^0$

Figure 3 shows the graph of q_r versus θ_2 where the values of θ_1 are from 0^0 to 90^0. The horizontal dashed line at $q_r = 0.8$ represents the critical flowrate ratio. For any θ_1, it can be seen that q_r approaches to the critical flowrate ratio except when $\theta_2 = 90^0$. To avoid over-flow in channel 1, both θ_1 and θ_2 cannot be 90^0.

From the results obtained, it can be observed that the right-angled bifurcation at one of the branches (either $\theta_1 = 90^0$ or $\theta_2 = 90^0$) would be efficient to reduce the amount of flowrates in channel 1 significantly. However, T-junction (when both θ_1 and θ_2 are 90^0) is not recommended.

5. Conclusions

This study provides insightful information for understanding of the open-channel flow and assists engineering design of river bifurcation. The mathematical model is derived based on continuity equation, momentum principle and some algebraic manipulations to predict the bifurcated river flowrates with different bifurcation angles. The model equation consists of Froude number and various important parameters such as bifurcation angles, width of channels, depth of flows and flowrates in branches of river. Thus, it can be applied for other rivers with different geometric properties. The analysis of the results reveals that the river flowrate after the bifurcated junction is below the critical flowrate if an appropriate bifurcation angles are considered. The implementation of right-angled bifurcation at Sungai Nenggiri can be an alternative action to mitigate flood.

In future study, the mathematical model for river bifurcation with different bifurcation angles can be investigated when the problem concerning the recirculation region is understood. Other interesting features that can be observed are the hydraulic jumps and the surface discontinuity. Furthermore, other minor factors affect the river bifurcation flowrate such as wall frictions and external forces that should be considered.

Nomenclatures

A: Cross-sectional area of the channel
C: Constant
F: Froude number
g: Gravitational acceleration
k: Side slope x flow depth to bottom width ratio
P: Pressure force
P_w: Wetted perimeter
Q: Flowrate
q_r: Flowrate ratio
b: Bottom width of the channel
B_r: Width ratio
T: Top width of the channel
U: Momentum transfer
V: Flow velocity
y: Flow depth
y_r: Flow depth ratio
ρ: Specific gravity
γ: Specific weight
θ: Bifurcation angles of channels 1 and 2

Subscripts

0: Main channel (Sungai Nenggiri)
1: Channel 1 (Sungai Nenggiri after bifurcation)
2: Channel 2 (Sungai Anak Nenggiri)
r: Ratio

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REFERENCES

[1] S. W. Law. Dividing Flow in Open Channel, Master Thesis, McGill University, Montreal, 1965.

[2] A. S. Ramamurthy, D. M. Tran, L. B. Carballada. Dividing Flow in Open Channels, Journal of Hydraulic Engineering, ASCE, Vol. 116, No. 3, 449–455, 1990.

[3] C. C. Hsu, C. J. Tang, W. J. Lee, M. Y. Shieh. Subcritical 90^0 Equal-Width Open-Channel Dividing Flow, Journal of Hydraulic Engineering, Vol. 128, No. 7, 716-720, 2002.

[4] I. M. H. Rashwan. Dynamic Model for Subcritical Dividing Flows in Open Channel Junction, Eight International Water Technology Conference, 511-520, 2004.

[5] N. L. Obasi, J. C. Agunwamba, N. Egbuniwe. Influence of Off-Take Angles on Flow Distribution Pattern at Concave Channel Bifurcation, Nigerian Journal of Technology, Vol.

27, No. 2, 46-57, 2008.

[6] A. Pandey, R. Mishra. Comparison of Flow Characteristics at Rectangular and Trapezoidal Channel Junctions, Journal of Physics Conference Series, Vol. 364, 1-11, 2012.

[7] G. Kesserwani, J. Vazquez, N. Rivière, Q. Liang, G. Travin, R. Mosé. New Approach for Predicting Flow Bifurcation at Right-Angled Open-Channel Junction, Journal of Hydraulic Engineering, ASCE, 662–668, 2010.

[8] D. A. Tholibon, J. Ariffin. Bifurcation Simulation Modeling Review, International Journal of Sciences: Basic and Applied Research, Vol. 8, No. 1, 45-50, 2013.

[9] A. Zahiri, A. A. Dehghani. Flow Discharge Determination in Straight Compound Channels using Anns, World Academy of Science, Engineering and Technology, Vol. 58, 12-15, 2009.

[10] S. A. Mirbagheri, M. Abaspour, K. H. Zamani. Mathematical Modeling of Water Quality in River Systems. Case Study: Jajrood River in Tehran – Iran, European Water, Vol. 27/28, 31-41, 2009.

[11] Y. Yang, T. A. Endreny, D. J. Nowak. Application of Advection-Diffusion Routing Model to Flood Wave Propagation: A Case Study on Big Piney River, Missouri USA, Journal of Earth Science, Vol. 27, No. 1, 9-14, 2016.