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A new higher-order RBF-FD scheme with optimal variable shape parameter for partial differential equation

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ABSTRACT

Radial basis functions (RBFs) with multiquadric (MQ) kernel have been commonly used to solve partial differential equation (PDE). The MQ kernel contains a user-defined shape parameter (ε), and the solution accuracy is strongly dependent on the value of this ε. In this study, the MQ-based RBF finite difference (RBF-FD) method is derived in a polynomial form. The optimal value of ε is computed such that the leading error term of the RBF-FD scheme is eliminated to improve the solution accuracy and to accelerate the rate of convergence. The optimal ε is computed by using finite difference (FD) and combined compact differencing (CCD) schemes. From the analyses, the optimal ε is found to vary throughout the domain. Therefore, by using the localized shape parameter, the computed PDE solution accuracy is higher as compared to the RBF-FD scheme which employs a constant value of ε. In general, the solution obtained by using the ε computed from CCD scheme is more accurate, but at a higher computational cost. Nevertheless, the cost-effectiveness study shows that when the number of iterative prediction of ε is limited to two, the present RBF-FD with ε by CCD scheme is as effective as the one using FD scheme.

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1. Introduction

The method of radial basis function (RBF) is an efficient technique in solving multidimensional interpolation problem due to the ease of implementation and use of directionally-independent kernel. In spite of many advantages, a highly ill-conditioned dense matrix needs to be solved especially for the case that involves a large number of nodes. A more practical approach is to use the local method, in which only a fixed number of neighboring nodes is taken into account. Application of this method results in a sparse linear system with smaller conditioning number.

The infinitely smooth RBF kernels such as the Gaussian and multiquadric (MQ) kernels introduced by Hardy [1] are known to give a more accurate solution as compared to the other types of kernels. However, the quality of the computed solution is strongly dependent on the introduced tuning shape parameter, ε, in the infinitely smooth kernels. The shape parameter also gives rise to a singularity problem as $\varepsilon \rightarrow 0$, in which the RBF linear system tends to be ill-conditioned.

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